On stable non-supersymmetric vacua at the bottom of cascading theories

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ABSTRACT: We consider a wide class of cascading gauge theories which usually lead to runaway behaviour in the IR, and discuss possible deformations of the superpotential at the bottom of the cascade which stabilize the runaway direction and provide stable non-supersymmetric vacua. The models we find may allow for a weakly coupled supergravity analysis of dynamical supersymmetric breaking in the context of the gauge/string correspondence.

KEYWORDS: Supersymmetric gauge theory, Gauge-gravity correspondence,
Supersymmetry Breaking.

Contents

1. Introduction and summary

A new approach to study the strongly coupled dynamics of gauge theories has been provided by their holographic correspondence with string theory on specific backgrounds (for reviews, see for instance [1–3], and [4, 5] for PhD thesis). In particular, with this approach one can in principle have access to properties of the theory, such as the full low-energy spectrum, which are beyond one's reach even when dealing with supersymmetric gauge theories.

Focusing on $\mathcal{N} = 1$ supersymmetric (SUSY) gauge theories, what on the other hand is within reach using purely gauge theoretic methods, is the information about the holomorphic quantities of the theory, in particular the number of isolated exact vacua, or the quantum moduli space of vacua. Paradoxically, it is possible by the same methods to argue that the theory has no supersymmetric vacua because of the non-perturbative dynamics [6, 7]. There are then two possibilities. Either one approaches a SUSY vacuum as some VEVs become infinite, which is referred to as having a runaway behaviour (the theory does not have a proper vacuum), or there is a stable non-supersymmetric vacuum, which we will refer to here as dynamical supersymmetry breaking (DSB). The latter option is obviously the favoured one when there is no moduli space at the classical level.

However, besides arguing that SUSY is broken at hierarchically small scales [8], we do not have the tools to really analyze quantitatively the physics around the vacuum. It would thus be extremely interesting to have at hand an example of gauge theory which displays DSB and which has a string/gravity dual. Keeping aside the usual problems related to the decoupling of the UV completion of the theory in the gravity dual, we would in principle be able to compute the spectrum of low-energy fields, their interactions, and so on.

A recent progress in the AdS/CFT correspondence was the discovery of new infinite classes of Sasaki-Einstein (SE) manifolds X^5 which, through considering type IIB string theory on $AdS_5 \times X^5$, correspond to some $\mathcal{N} = 1$ four-dimensional superconformal gauge theories, engineered placing a bunch of N D3 branes at the tip of the CY cone over X^5. These new manifolds were dubbed $Y^{p,q}$ [9], $L^{p,q,r}$ [10, 11] and $X^{p,q}$ [12] (of the latter the explicit metrics are not known). The dual quiver gauge theories were constructed and many checks of the correspondence were carried out [13 – 18].

A subsequent progress, which is more important to our purposes here, came adding fractional branes to these systems. Fractional branes are usually meant to break conformal invariance and move towards more realistic and dynamically interesting theories. In particular, they trigger a renormalization group flow which takes the form of a cascade of Seiberg dualities reducing the ranks of the gauge groups as we flow towards the IR, as first discussed for the conifold case [19 – 21]. The cascade goes on until for some gauge group(s) the number of flavours equals the number of colours. At this point the quantum moduli space of the corresponding gauge group(s) gets modified. The theory confines along the baryonic branch and the full theory is reduced by one (or more) gauge group factor. The theory stops cascading as the bottom of the cascade has been reached. If one looks at the structure of the gauge theory within this energy range, it is as if regular branes have completely disappeared and only fractional branes have survived. The IR dynamics of the theory is then determined solely by the fractional branes. For a detailed review on the physics of the cascade we refer the reader to the beautiful paper [21].

Contrary to the case of the conifold [20], where the cascade ends up with confining SUSY vacua, fractional branes on these newly found SE manifolds end up at the bottom of the cascade with the generation of an Affleck-Dine-Seiberg (ADS) superpotential, and hence generically break supersymmetry. It has by now become clear that there exist three classes of fractional branes in string theory, according to the different IR dynamics they induce in the dual gauge theory [22]: *i)* $\mathcal{N} = 2$ branes, which correspond to branes wrapped on cycles that are not located at a point-singularity but rather on a curve singularity and hence have a moduli space and generate $\mathcal{N} = 2$ supersymmetric dynamics; *ii) deformation* branes, which trigger confinement and lead to supersymmetric vacua; *iii) supersymmetry breaking* (SB) branes, which generate an ADS superpotential and might lead to stable non-supersymmetric vacua or runaway behavior in the dual gauge theory.

In all the cases with SB branes that have been studied a runaway direction develops [23, 22, 24 – 27]. Therefore, the possibility of getting fully stable non-supersymmetric vacua, in the context of the gauge/string correspondence, remains an open challenge. [1]

In this paper, we address this issue and show how in some cases simple deformations of the tree level superpotential can change the IR dynamics in such a way that stable non-supersymmetric vacua do arise. This is not generic and holds only in very specific cases. This pairs with the fact that DSB models with stable vacua arise only for very specific choices of gauge groups and matter content, while generically one expects either

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[1]The possibility of meta-stable long-lived vacua was pointed out recently for SQCD with massive [28] and massless [29] flavors. These models find a nice realization into flavoured versions of AdS/CFT [29]. See also [30] for more work on meta-stable DSB.

supersymmetric vacua or runaway behaviour. It would be nice to understand this property directly from string theory. We perform a pure field theory analysis, and do not attempt to answer this question here, though some comments are given in the discussion section.

Our analysis suggests that a generic, almost model-independent pattern for DSB emerges. We consider, as explicit examples, the gauge theories on fractional branes placed at the tip of the complex cone over toric del Pezzo surfaces, dP_k. In fact, these are related to the SE manifolds recently found. In particular, the complex cone over the first del Pezzo is the real cone over $Y^{2,1}$ [13], and the complex cone over the second del Pezzo is the real cone over $X^{2,1}$ [12]. Hence the corresponding dual gauge theories are also the same (for earlier works on regular branes at del Pezzo conical singularities, see [31, 32]). The metrics for the CY cone over the del Pezzo surfaces are not known, but for dP_1. However, the toric diagrams are known. This is enough to derive the dual gauge theory [33 – 35]. Support for a cascading dynamics for the gauge theories of fractional branes at del Pezzo's conical singularities was given by supergravity analysis in [36, 37]. For a gauge theory analysis confirming these expectations we refer to [38, 23, 22, 24]. In what follows, we will always assume that a cascade takes place and analyze the gauge theory directly at the last step, i.e. in the deep IR.

The paper is organized as follows.

In section 2 we briefly review known results about the dP_1 case and partially extend them. We show, in particular, that stable non-supersymmetric vacua arise only when perturbing the superconformal fixed point by a single fractional brane while for a generic number M of fractional branes no deformations are possible which lift all classical flat directions and cure the runaway behaviour. We also argue that it is not possible to obtain a stable non-supersymmetric vacuum in the full class of $Y^{p,q}$ manifolds, independently on the number M of fractional branes.

In section 3 we consider the case of the second del Pezzo surface. There are two kinds of fractional branes here: deformation branes and SB branes. Similarly to the previous case, the latter induce a runaway behaviour. The generic situation is having P deformation branes and K SB branes. Similarly to the dP_1 case, the addition of one (and only one) SB brane, $K = 1$, allows for a deformation of the superpotential which stabilizes the runaway direction and provides a stable non-supersymmetric vacuum at finite field VEVs. There is a crucial difference with respect to the dP_1 case, though. The DSB model we find here allows for a large number of deformation branes. This makes it possible to perform a probe analysis of a single SB brane in the weakly coupled and well-behaved background generated by the deformation branes. We do not pursue this analysis here, nevertheless in principle a dual supergravity description is possible in this case.

Section 4 contains the analysis for dP_3. The story repeats: generically, it is not possible to find deformations of the superpotential that stabilize the runaway direction and get a non-supersymmetric vacuum. However, for a specific choice of fractional branes (and only for such a choice) we recover a model very similar to the one working for dP_2. Once again, a large number of deformation branes may support a smooth and weakly coupled background where to perform a probe analysis.

Section 5 contains a discussion for other del Pezzo toric varieties, the so-called pseudo
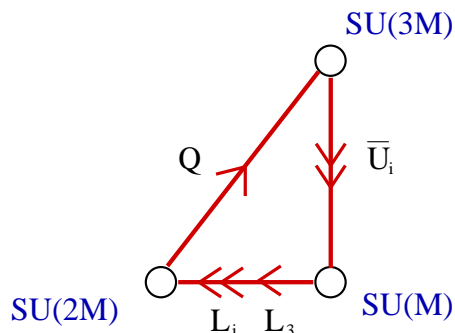
Figure 1: Quiver of the dP_1 theory for M fractional branes.

del Pezzo's [39]. We focus for definiteness on PdP_4 and show that similar phenomena to those occurring for dP_2 and dP_3 hold.

We conclude in section 6 with a discussion. Our analysis indicates that a generic pattern for DSB does emerge. More precisely, the actual models of DSB always turn out to be a minimally modified version of the $SU(N) - SU(2)$ model [40]. Unfortunately, we cannot yet provide a direct string theory explanation of this result, but the possibility of performing a probe analysis of the DSB phenomenon makes it worth trying to find KS-like solutions corresponding to these geometries.

2. Stable vacua at the bottom of the dP$_1$ cascade

Let us start by analyzing the simplest case, namely the dynamics of a set of M fractional D3 branes on the complex cone over the first del Pezzo surface, dP_1. In this case there is only one type of fractional brane which is of the SB type [23, 22, 24]. The IR dynamics of this model has been analyzed in detail in the literature, showing that it displays a runaway behaviour along a baryonic flat direction. In [25], it was also noted that for a single fractional brane ($M = 1$) the runaway direction can in fact be converted into a stable non-supersymmetric vacuum via some modification of the tree level superpotential. The theory at the bottom of the cascade gets essentially reduced to the 3-2 model of [7]. Here, we briefly repeat the analysis to set up the framework for the more general cases discussed later.

The theory at the bottom of the cascade is depicted in the quiver in figure 1.

There is in addition a tree-level superpotential

$$W_{\text{tree}} = h \, Q \, \bar{U}_i L_j \, \epsilon^{ij} \qquad i = 1, 2 \,, \tag{2.1}$$

while the R-charges of the chiral superfields are reported in the table below (the $\overline{U}_i$'s and L_i's are both in a doublet of the global SU(2) and hence must have the same R-charge).

This theory has a number of classical flat directions. It is easy to see that the F-flatness conditions derived from the superpotential (2.1) set all the invariants involving the fields Q and $\overline{U}_i$ to zero. However there are a number of "baryonic" invariants, schematically

	Q	$\overline{U}_i$	L_i	L_3
U(1)$_R$	-1	0	3	-1

written as

$$\mathcal{B}_{i_1 \ldots i_k} = \epsilon_{2M} \epsilon_M \epsilon_M (L_i)^{2M-k} (L_3)^k \ , \qquad k = 0, \ldots, M \ , \qquad (2.2)$$

where ϵ_{2M} and ϵ_M are the Levi-Civita tensors of SU(2M) and SU(M), respectively. One can actually see that if we consider the $L_a = (L_i, L_3)$ to be in a triplet **3** of a global (accidental) SU(3), then all the invariants above fall into one SU(3) representation which is the Mth symmetric product of the $\overline{\mathbf{3}}$, of dimension $\frac{1}{2}(M+1)(M+2)$. This representation splits into a sum of SU(2) symmetric representations with k indices, for $k = 0, \ldots, M$, corresponding to the $\mathcal{B}_{[k]}$ above.

As explained in [25], at a generic point of the moduli space spanned by the $\mathcal{B}_{[k]}$'s, the superfields Q and $\overline{U}_i$ get a mass, and the determinant of the mass matrix is proportional to $\mathcal{B}_{[0]}$ (the baryon without L_3's, which is the global SU(2) singlet). Hence, via the matching of scales and the generation of a non-perturbative superpotential for the confining SU(3M) gauge group, we have a runaway behaviour along the direction parameterized by $\mathcal{B}_{[0]}$.[2]

The above reasoning is true for any number M of fractional branes. We can now ask whether or not it is possible to lift the classical flat directions by deforming the tree level superpotential with the addition of baryonic couplings. Indeed, lifting the classical flat directions is a first step towards finding a model which realizes DSB.

Another crucial ingredient for obtaining DSB is the following. A well known criterion [6, 7] for the possibility of having a stable non-supersymmetric vacuum in a supersymmetric gauge theory is the presence of a non-anomalous R-symmetry which is spontaneously broken by quantum effects. If a theory realizes this condition and does not have classical flat directions, one expects the full quantum theory to develop a stable non-supersymmetric vacuum.[3]

The theory at the bottom of the dP_1 cascade, figure 1 and superpotential (2.1), has indeed a non anomalous R-symmetry, independently of the value of M. When one aims to suitably deform the theory in order to lift the classical flat directions, it is crucial to check that the extra contributions to the tree level superpotential respect this symmetry. In other words, the operators one wants to add to W_{tree} must have R-charge 2.

The R-charge of the baryonic operators $\mathcal{B}_{[k]}$ is simply

$$R(\mathcal{B}_{[k]}) = 6M - 4k \geq 2M. \qquad (2.3)$$

The smallest R-charge is provided by the baryon with most L_3's, in the largest SU(2) representation, i.e. $\mathcal{B}_{[M]}$. This R-charge can be equal to 2 only if $M = 1$. Hence, only

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[2]The Kähler potential, which is approximated by the classical one for large enough VEVs, can be shown to be consistent with this conclusion.

[3]Indeed, the well-known argument is that SUSY must also be broken since there is no room for a flat non-compact direction parameterized by a putative scalar superpartner of the Goldstone boson associated to the breaking of the R-symmetry.
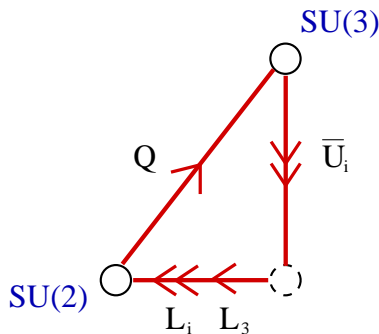
Figure 2: Quiver of the dP_1 theory for one fractional brane. The dashed node represents an "SU(1)" factor, which plays no rôle in the IR.

in the presence of a single fractional brane we expect to have a deformation of W_{tree} that leads to DSB. In [25] it was shown that this is really the case. Indeed, the superpotential

$$W_{\text{def}} = W_{\text{tree}} + \alpha \mathcal{B}_1 = h \, Q \, \bar{U}_i L_j \, \epsilon^{ij} + \alpha L_1 L_3 \tag{2.4}$$

preserves the R-symmetry and effectively reduces the theory to the 3-2 model of [7], which displays DSB. The theory with $M = 1$ is reported in figure 2

It can be checked that for $M > 1$ it is not possible to have DSB. For $M = 2$, a straightforward but tedious computation reveals that, if one succeeds in lifting all the classical flat directions, the effect of the quantum dynamics is just to displace the SUSY vacua at finite distance. For $M > 2$, one can easily see that at the classical level it is impossible to write simple couplings (i.e. linear in the baryonic operators) which lift all the flat directions. Hence at the quantum level we will eventually get either runaway behaviour or moduli spaces of SUSY vacua.

For future reference let us finally notice that in what discussed above, a crucial ingredient for being able to write the extra term in the superpotential (2.4), was to have bi-fundamental fields between an SU(2) factor and an "SU(1)" factor.[4] This basically allows one to "turn around" the arrow in forming a quadratic gauge invariant.

To conclude, the main lesson of the dP_1 case is that, while for any number M of fractional branes the IR dynamics is runaway, only for $M = 1$ it is possible to cure this behaviour and achieve DSB. This is done by adding a suitable term to the tree level superpotential.

We know from general considerations in the gauge/gravity correspondence that having only one fractional brane is the opposite regime with respect to the one for which the supergravity approximation is really under control. In the present example, we cannot even argue that the single fractional brane could be treated in the probe approximation, since the same fractional brane is at the same time also responsible for the confining

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[4]In this paper we consistently neglect the presence of possible U(1) gauge factors on every node. Besides the diagonal one which decouples completely, we will assume that all the others get a mass through a GS mechanism, whether or not there are anomaly free combinations.

dynamics at the SU(3) node, which should correspond in some way to a deformation of the dual geometry removing the singularity at the apex of the cone. This dynamics would clearly be missed in a probe approximation. We will be able to circumvent this difficulty by considering the theory based on dP_2 (and higher del Pezzo's).

2.1 Comments on the $Y^{p,q}$ family

As already reminded, the complex CY cone over dP_1 is the same as the real CY cone over $Y^{2,1}$ [13], this being the simplest instance of the infinite $Y^{p,q}$ family. The corresponding gauge theory duals have been widely studied in the last two years and all results, so far, indicate that the basic physical properties are quite the same, both in the conformal and in the non-conformal cases, regardless the specific value of p and q. Hence, one could ask if a story similar to the one just discussed holds for any $Y^{p,q}$. In what follows, we argue this is not the case.

The physics of the gauge theory dual to the $Y^{p,q}$ manifolds with fractional branes is believed to be a cascading theory, as it is the case for $Y^{2,1}$. There is a single type of fractional branes leading to a unique non-conformal quiver [14]. Similarly to $Y^{2,1}$ the gauge theory at the bottom of the cascade has a classically flat baryonic runaway direction [26]. [5]

The more p and q are increased, the more the analysis gets involved. However, explicit computations for $Y^{3,1}$ and $Y^{3,2}$ show that, independently on the number of fractional branes, the runaway direction cannot be lifted. Analyzing the structure of the corresponding quivers one sees that also for $M = 1$ there are not enough matter fields charged under an SU(2) and an "SU(1)" factor, this being a crucial ingredient to make a DSB deformation possible. Furthermore, there are several mesonic trace operators absent from the superpotential which are difficult to lift. This is a common feature for the whole $Y^{p,q}$ family. Hence, we expect that for p and q not being equal to 2 and 1, respectively, it is not possible to deform the fractional brane gauge theory and get stable non-supersymmetric vacua [41].

## 3. Stable vacua at the bottom of the dP_2 cascade

As we have seen in the previous section, in the dP_1 case we have found only one instance where DSB could be provoked. In a sense, it would be nicer to have a whole class of such models, where by tuning one parameter we can place ourselves in a regime more suitable to study the gravitational counterpart. For this reason, we turn to study fractional branes at the tip of the complex cone over the second del Pezzo surface, dP_2. This system was widely studied recently, see [22, 42].

In dP_2, there are two kinds of fractional branes, one of deformation type and one of supersymmetry breaking type. In figure 3 we show the quiver diagram one gets at the end of the cascade with P deformation branes and K SB branes. The cascade in this case is more subtle than in the dP_1 case, for instance it is self-similar only after more than one

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[5]The details of the cascade are fully understood only for $q = 1, p - 1$. However, the authors of [26] have given convincing arguments for the runaway nature of the whole family.
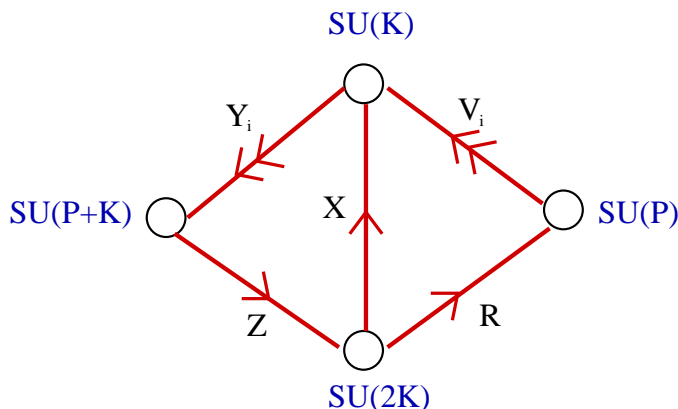
Figure 3: Quiver of the dP_2 theory for generic P and K.

step of Seiberg dualities, and the number of steps after which the pattern repeats itself depends on the relative size of P and K. However, one can check that in all the cases of interest one indeed finds the quiver of figure 3 at the bottom of the cascade, with the relevant values of P and K.

The tree level superpotential inherited from the conformal quiver is

$$W_{\text{tree}} = -Y_1 Z X + Y_2 Z R V_1 . \tag{3.1}$$

Clearly, there is a number of different cases we can consider. Besides the generic case, where we have 4 gauge groups at the end of the cascade, we have particular cases with less gauge groups if we have 0 or 1 branes of one or both kinds.

It is easy to see that adding only deformation branes ($K = 0$), at low energies we basically get two decoupled $SU(P)$ SYM theories, together with baryonic branches which can be seen analyzing the next-to-last step of the cascade, as in the conifold case [20].

On the other hand, in the presence of only SB branes ($P = 0$), we end up with a triangular quiver, as in figure 4.

This can be shown to have a runaway behaviour along the same lines as in the triangle at the bottom of the dP_1 cascade discussed in the previous section. The R-charges are listed in the table below.

	X	Z	Y_1	Y_2
$\text{U}(1)_R$	-1	-1	4	0

The difference here is that in this case the R-charges of the baryonic invariants $(Y_1)^k$ $(Y_2)^{K-k}$ which parameterize the classical flat directions, are $R_k = 4k$, and hence never equal to 2, no matter which number of fractional branes we have. Therefore, terms which could lift the flat directions would also break the R-symmetry and possibly lead to SUSY vacua, not to DSB. This can be checked by direct computations in a few low rank cases.
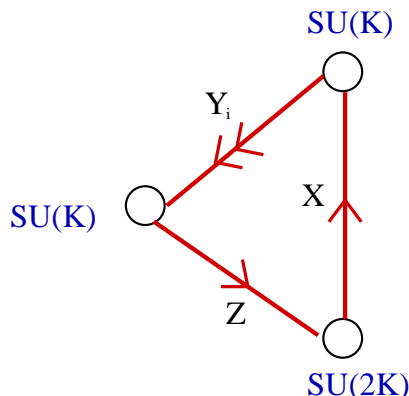
Figure 4: Quiver of the dP_2 theory for generic K and $P = 0$.

In a similar fashion, also the cases with generic P and K, and with K generic[6] and $P = 1$ can be shown not to allow the lifting of classical flat directions while preserving the R-symmetry.

We thus turn to the only remaining case, that is the one where we have a (possibly large) number P of deformation branes, to which we add a single $K = 1$ SB brane. As we have seen before, we found DSB in the dP_1 case by deforming the theory for a single fractional brane to the well-known 3-2 model. Here we will also be in a setup where a known model of DSB is recovered. Let us then go through a more detailed analysis of this model.
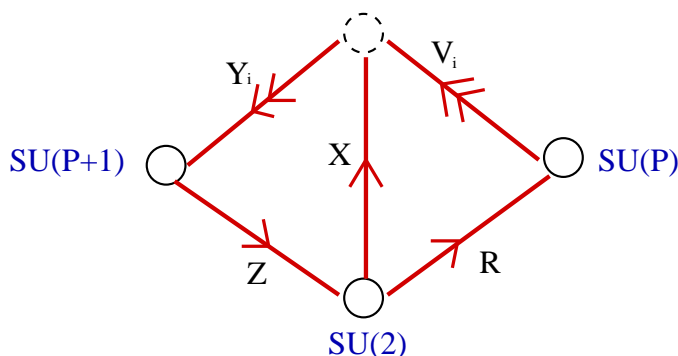
The simplest generalization of the 3-2 model of [7] is to replace the SU(3) group by SU(N) [45], with N odd to prevent the SU(2) global anomaly. Nothing really differs from the 3-2 model, except for the fact that the gauge dynamics remains strongly coupled in the IR because an unbroken SU($N-2$) gauge group survives. A slight modification of this model was presented in [40]. The SU(2) group has N extra doublets instead of only one. Still, it turns out that DSB is possible again only if N is odd. The reason is that one is led to add mass terms for all but one of the extra doublets, but these mass terms are really "baryonic" terms of SU(2) and hence the mass matrix can have a single zero eigenvalue only if it has odd dimensions.[7]

We now come to the analysis of the model at the base of the cascade for dP_2 with P deformation fractional branes and $K = 1$ SB fractional branes. The gauge group consists of 3 nodes: SU($P + 1$) × SU(2) × SU(P), and the matter content is given in the quiver in figure 5 with the charge assignment in the table below (note that we have used that $\mathbf{2} = \overline{\mathbf{2}}$, and we have included R-charges that will be discussed later on).

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[6]If $K = 1$ and $P = 1$, we have two SU(2) gauge factors, each with 4 doublet matter fields and additional singlet fields. (Un)luckily, the superpotential is not of IYIT type [43, 44] and the usual R-charge considerations show that this case should not lead to DSB.

[7]In [40] the SU(N)−SU($N-2$) model is also analyzed, which is obtained by performing a Seiberg duality on the SU(2). Of course the conclusions about the IR behavior must be the same, but the description is in a sense under better control (i.e. weakly coupled).

	SU($P+1$)	SU(2)	SU(P)	U(1)$_R$
Y_1	$\overline{\mathbf{P+1}}$	$\mathbf{1}$	$\mathbf{1}$	$4-2P$
Y_2	$\overline{\mathbf{P+1}}$	$\mathbf{1}$	$\mathbf{1}$	2
Z	$\mathbf{P+1}$	$\mathbf{2}$	$\mathbf{1}$	-2
X	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$2P$
R	$\mathbf{1}$	$\mathbf{2}$	$\overline{\mathbf{P}}$	2
V_1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{P}$	0
V_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{P}$	$-2P$

Figure 5: Quiver of the dP_2 theory for generic P and $K = 1$. The dashed node represents an "SU(1)" factor, which plays no rôle in the IR.

The expected dimension of the moduli space (in the absence of a tree level superpotential) is given as follows. We have $4(P+1) + 4P + 2 = 8P + 6$ matter fields, and by giving VEVs to all of them we are higgsing $(P+1)^2 - (P-1)^2 + 3 + P^2 - (P-2)^2 = 8P - 1$ gauge fields, leaving a total of 7 classical flat directions. The gauge invariants we can build are

$$a_i = Y_i^a Z_a^\alpha X^\beta \epsilon_{\alpha\beta}, \qquad b_{ij} = Y_i^a Z_a^\alpha R_m^\beta V_j^m \epsilon_{\alpha\beta}, \qquad c = Z_a^\alpha Z_b^\beta Y_i^a Y_j^b \epsilon_{\alpha\beta} \epsilon^{ij},$$

$$d_i = X^\alpha R_m^\beta V_i^m \epsilon_{\alpha\beta}, \qquad e = R_m^\alpha R_n^\beta V_i^m V_j^n \epsilon_{\alpha\beta} \epsilon^{ij} . \tag{3.2}$$

Note that the invariants d_i are cubic only because the middle node is SU(2) and not of higher rank (in general they are baryons involving $3K$ fields). The above invariants are 10, but we also have the 3 constraints,

$$cd_i = a_j b_{ki} \epsilon^{jk}, \qquad ce = b_{ij} b_{kl} \epsilon^{ik} \epsilon^{jl}. \tag{3.3}$$

The theory at the base of the cascade comes with the classical tree level superpotential

$$W_{\text{tree}} = -a_1 + b_{21} = -Y_1 Z X + Y_2 Z R V_1 . \tag{3.4}$$

We can easily work out the F-flatness conditions, and find that the invariants a_i, b_{ij} and c are set to zero while the d_i and e are left undetermined.

Thus the latter are the light fields at low energies, and at a generic point of the moduli space the fields X, R and V_i get VEVs. This in particular implies, through (3.4), that the fields Y_i and Z get masses, with a mass matrix which is such that $\det m \propto d_1$. At low energies, the gauge group is broken to $\mathrm{SU}(P+1) \times \mathrm{SU}(P-2)$ and the scale matching goes as follows

$$\Lambda_{P+1,\mathrm{low}}^{3P+3} = h d_1 \Lambda_{P+1}^{3P+1}, \quad \Lambda_{P-2,\mathrm{low}}^{3P-6} = \frac{\Lambda_P^{3P-2}}{e}, \tag{3.5}$$

where h is the coupling of the quartic term in W_{tree}, reintroduced here for dimensional reasons. The effective superpotential is thus

$$W_{\mathrm{eff}} = (P+1) \left(h d_1 \Lambda_{P+1}^{3P+1} \right)^{\frac{1}{P+1}} + (P-2) \left(\frac{\Lambda_P^{3P-2}}{e} \right)^{\frac{1}{P-2}}. \tag{3.6}$$

It is clearly runaway, both in d_1 and in e. The Kähler potential for the light modes can be computed, and it is given by exactly the same expression as the one given in [7] for the 3 moduli of the 3-2 model. Note that the model at hand is similar to two $\mathrm{SU}(N) - \mathrm{SU}(2)$ models sharing the same $\mathrm{SU}(2)$ node. However, it turns out P is not constrained to be even or odd.

As in the 3-2 model, it seems now plausible that adding a tree level term including d_2 should lead to DSB. Though it would be nice to check this using (3.6) and the Kähler potential, it is easier to analyze the theory using the fundamental fields and their F-flatness conditions. We thus consider

$$W_{\mathrm{def}} = W_{\mathrm{tree}} + d_2 = -Y_1 Z X + Y_2 Z R V_1 + X R V_2. \tag{3.7}$$

Now the classical F-conditions set all the invariants to zero, so that we are in a situation where there is no classical moduli space. A first indication that we should have DSB is that there is a non-anomalous R-symmetry (whose charges are given in the table) which is preserved by the above superpotential. Since (3.6) will push some invariants to have non zero VEVs, the R-symmetry will be spontaneously broken, thus SUSY must be broken, too.

We now write the effective superpotential including the non-perturbatively generated ADS superpotential for both $\mathrm{SU}(P+1)$ and $\mathrm{SU}(P)$ gauge groups

$$W_{\mathrm{eff}} = W_{\mathrm{def}} + (P-1) \left(\frac{\Lambda_{P+1}^{3P+1}}{Z^2 Y_1 Y_2} \right)^{\frac{1}{P-1}} + (P-2) \left(\frac{\Lambda_P^{3P-2}}{R^2 V_1 V_2} \right)^{\frac{1}{P-2}}. \tag{3.8}$$

The F-flatness conditions with respect to X, V_i and Y_i impose that $a_2 = b_{11} = b_{12} = b_{22} = c = d_1 = e = 0$ and that $a_1 = b_{21} = d_2 \propto c^{-1/(P-1)} \to \infty$. We now impose the F-flatness with respect to R_m^α and get

$$(Y_2 Z)^\alpha V_1^m + X^\alpha V_2^m = \left(\frac{\Lambda_P^{3P-2}}{e} \right)^{\frac{1}{P-2}} \frac{1}{e} \left[(RV_1)^\alpha V_2^m - (RV_2)^\alpha V_1^m \right]. \tag{3.9}$$

Multiplying by $(X^\gamma \epsilon_{\alpha\gamma})(R_n^\beta V_2^n R_m^\delta \epsilon_{\beta\delta})$, we get

$$a_2 e = \left(\frac{\Lambda_P^{3P-2}}{e}\right)^{\frac{1}{P-2}} d_2 \,, \tag{3.10}$$

which is an inconsistency since the l.h.s. is supposed to vanish while the r.h.s. should blow up. Hence, the F-flatness conditions have no solution, even for infinite VEVs. We conclude that this model displays true DSB and a stable non-supersymmetric vacuum.

Again, the coupling that we had to introduce to stabilize the runaway behaviour uses the fact that, due to the SU(2) group, we can turn around the arrow for the field X, and the coupling is baryonic in this sense.

Let us stress the basic difference between the present case and the dP_1 case. Here we have P deformation branes which provide the non-perturbative correction to the super-potential. This translates to the smooth deformation of the gravity dual, which should be (for $K = 0$) a KS-like geometry. Then comes the addition of the single SB fractional brane, which could in principle be treated in the probe approximation, also because all the effects it has on the dynamics are through tree level couplings in the superpotential and the appearance of an SU(2) gauge group which is IR-free for sufficiently large P. Hence, we expect that it could be possible to see both the runaway behaviour, with the original W_{tree}, and a stable non-supersymmetric configuration, with W_{def}, on the world-volume of the probe fractional brane in the KS-like background produced by the $P \gg 1$ deformation branes through geometric transition.

3.1 Comments on the $X^{p,q}$ family

The second del Pezzo surface can be obtained as a blow-up of the first del Pezzo. The field theory dual of this geometric operation was shown in [39] to correspond to an unhiggsing procedure, i.e., the superconformal gauge theory dual to dP_2 can be obtained by unhiggsing from that of dP_1. Based on this, in [12] a new family of toric singularities, dubbed $X^{p,q}$, was constructed and the corresponding dual gauge theories derived. A SE manifold $X^{p,q}$ is related to $Y^{p,q}$ right in the same way as dP_2 is related to dP_1. And a similar relation holds for the corresponding dual gauge theories. For instance, the gauge theory dual to $X^{p,q}$ has one gauge group more than the corresponding $Y^{p,q}$ theory.[8]

The $X^{p,q}$ theories admit two kinds of fractional branes, similarly to dP_2, one of them being of the SB type and leading, in the IR, to a runaway behaviour. Once again, one could ask if the runaway can be stabilized by some suitable deformation of the superpotential. Again, we suggest the answer to be negative. Explicit computations for some simple cases, and general considerations similar to those holding for $Y^{p,q}$, suggest that as long as p and q increase, the number of classical flat directions increases faster then the number of possible "stabilizing deformations" to be added to the superpotential [41].

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[8]To find the explicit metrics of the $X^{p,q}$ is still an open problem.
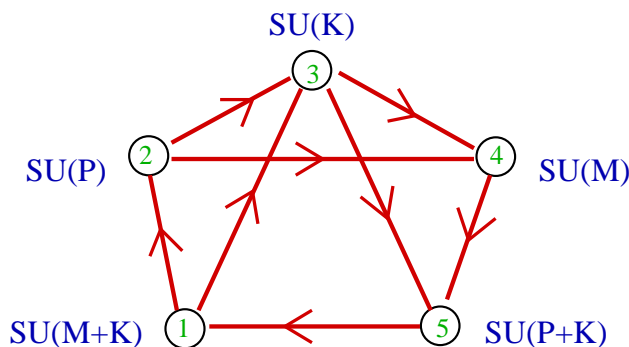
Figure 6: Quiver of the dP_3 theory for generic M, P and K.

4. Stable vacua at the bottom of the dP$_3$ cascade

One might ask if the pattern of DSB we have described for dP_2 goes through as we go on with higher del Pezzo surfaces, which are in fact all related by subsequent blow-ups. The analysis for the next del Pezzo, dP_3, is straightforward. This theory and the corresponding cascade was already studied in the literature. We refer to [36, 38, 22] for more details.

The dP_3 theory admits three different independent fractional branes, all of the deformation type. Two of them, similarly to the deformation brane of the dP_2 case, lead in the IR to two decoupled nodes. The third one leads to a triangle quiver with the corresponding cubic gauge invariant being present in the tree level superpotential (resulting, however, in the same confining dynamics). The theory at the bottom of the cascade, for generic values of the three fractional branes, M, P and K, respectively, is clearly more complicated. It is depicted in figure 6 and has a tree level superpotential

$$W = X_{13}X_{35}X_{51} - X_{12}X_{24}X_{45}X_{51} \ . \tag{4.1}$$

As shown in [22] these deformation branes are not mutually supersymmetric. More precisely, for generic M and P but $K = 0$, the cascade does end in a supersymmetric confining theory, while, whenever K and M (and/or P) are simultaneously different from zero, then an ADS superpotential is generated (this should be clear from the quiver in figure 6). Take, for instance, $P = 0$ and generic M and K. In this case the theory reduces to four gauge groups with group 1 having $N_F < N_C$ and a superpotential with only the cubic coupling

$$W = X_{13}X_{35}X_{51} \ . \tag{4.2}$$

The dynamically generated ADS superpotential leads to runaway. Indeed, for generic values of M, P and K there are plenty of classical flat directions, which are very difficult to lift altogether. A general analysis based on possible R-charge preserving superpotential couplings singles out only one possibility which allows for a deformation which lifts all classical flat directions. This is for generic M and $P = K = 1$ (or equivalently generic P and $M = K = 1$) see figure 7.
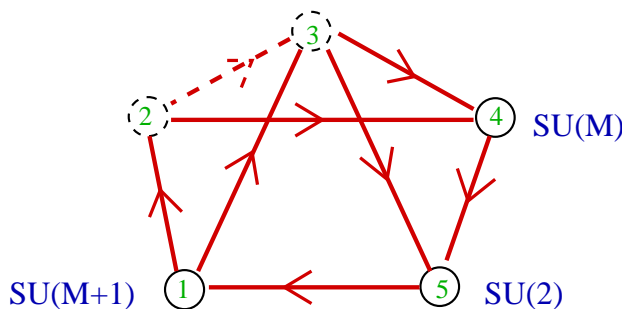
Figure 7: Quiver of the dP_3 theory for generic M and $P = K = 1$. The dashed nodes represent "SU(1)" factors while the dashed line a bi-fundamental field which completely decouples in the IR and plays no rôle. This theory is essentially the same as the one for dP_2, see figure 5.

Note that since the "SU(1)" gauge factors actually do not exist, the IR theory is nothing but the one at work for the dP_2 case, figure 5. Hence, the analysis is exactly the same as the one in the previous section, as well as the conclusions. This seems to indicate that some sort of unique picture emerges from (toric) del Pezzo's. The analysis in the next section will support this claim.

Similarly to the case of the first and the second del Pezzo surfaces, one could argue the existence of a full family of new (toric) SE manifolds, call them $Z^{p,q}$, with three different kinds of fractional branes, whose dual gauge theories should be obtained by unhiggsing from those dual to $X^{p,q}$. In fact, it should be possible to construct such new dual pairs, along the same lines of [12] for $X^{p,q}$, using toric geometry. We expect that similar conclusions as those for the Y and X families hold for the Z family, i.e. no possible stabilization of the otherwise runaway behaviour whenever $p > 2$, $q > 1$.

5. Higher (pseudo) del Pezzo's

By further unhiggsing one can go up in the number of gauge groups and get other dual toric varieties by the geometric dual blow-up procedure [39]. These were dubbed Pseudo del Pezzo, PdP_k, not to be confused with actual del Pezzo's which, for $k > 3$, are not toric. We consider here, as an example, the PdP_4 theory. This model admits four different kinds of fractional branes [22]. Three are deformation branes and pair exactly those of dP_3. The last is a $\mathcal{N} = 2$ brane (it leads to a triangle quiver in the IR with the corresponding cubic gauge invariant *not* being present in the tree level superpotential). For generic numbers M, P, K and L of these fractional branes, respectively, the quiver at the bottom of the cascade is as in figure 8,

with a superpotential

$$W = X_{12}X_{24}X_{45}X_{51} - X_{63}X_{34}X_{45}X_{56} + X_{35}X_{56}X_{62}X_{23} - X_{13}X_{35}X_{51} \ . \tag{5.1}$$

Suppose to consider no $\mathcal{N} = 2$ branes first, i.e. $L = 0$. It is easy to see that the quiver in figure 8 reduces exactly to that of dP_3, figure 6 (just drop node 6 and the corresponding
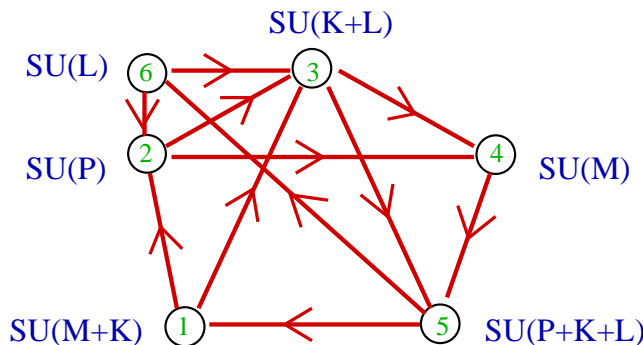
Figure 8: Quiver of the PdP_4 theory for generic M, P, K and L.

matter fields) and the superpotential (5.1) reduces to the superpotential (4.1). Therefore the analysis in this case is the same as before: the *only* possibility which allows for a stabilization of the runaway behaviour is taking M generic (and possibly large) and $P = K = 1$, as we did in the case of dP_3. Once again, as far as DSB is concerned, the physics resembles that of dP_2: only in one specific case DSB is possible, providing a model resembling very much the $SU(N) - SU(2)$ model.

We are left to consider the case where $L \neq 0$. The analysis gets clearly more involved and we limit here to few basic observations. Depending on the relative value of M, P, K and L there are indeed ADS superpotential terms being generated and possible DSB. However, the presence of L makes the appearance of an extra gauge invariant, $X_{35}X_{56}X_{63}$, which is *not* present in the superpotential. Hence, for $L \neq 0$, on top of the flat baryonic directions there will also be additional mesonic flat directions, which makes it even more unlikely to find DSB by adding simple terms to the superpotential.

6. Discussion

In this paper we have presented a few instances of cascading quiver gauge theories where, by deforming the tree level superpotential at the bottom of the cascade by simple "baryonic" terms, we could find genuine dynamical supersymmetry breaking in a stable vacuum. The pattern which seems to emerge is that we can find one such instance in each set of quiver gauge theories dual to branes on the complex cone over del Pezzo surfaces. Except for the case of dP_1, it is possible to have a large number of deformation branes and thus treat the addition of the other branes which lead to SUSY breaking in the probe approximation.

We have also argued that we do not expect the same mechanism to work in the infinite families of quiver gauge theories which are generalizations of the ones corresponding to del Pezzo surfaces, as the $Y^{p,q}$ and $X^{p,q}$. In this line of reasoning, it would be interesting to use more systematic methods to address these questions in general [41], taking also into account the relationship among these different theories by higgsing and unhiggsing [39]. Though we expect that the outcome of this more general study will be more of the kind of a "no-go" theorem except for the known cases, there is still the possibility that other kinds of

models of DSB would appear to be relevant. With respect to this possibility, we think that it is nevertheless likely that it will always be necessary to deform in some way the theory in order to get DSB. Indeed, though a definite theorem still does not exist, there is convincing evidence [26] that the generic behaviour when fractional branes of any kind are added to a conformal quiver gauge theory is that we end up at the end of the cascade with either SUSY vacua, or a runaway behaviour. This is essentially because "baryonic" invariants are always present at the bottom of the cascade, and they are always left as classical flat directions by the tree level superpotential inherited from the conformal quiver.

It would also be interesting to extend these ideas to brane systems probing non-toric geometries, such as for instance the $dP_{n>3}$ surfaces, which have remained elusive to this point.

We should pay a closer attention to the terms that we add to the tree level superpotential in order to have DSB, terms that we generically dub "baryonic". These terms can be written because at the bottom of the cascade an SU(2) gauge factor arises and arrows can be reversed. An interesting question is to see how these terms would look like if we go up the cascade, or in other words what their origin can be in the theory far from the IR, where a large (effective) number of regular branes is present. Though performing a detailed analysis is beyond the scope of the present work, we expect these operators to correspond to (combinations of) baryonic-like operators, of which there are plenty at a generic step in the cascade. Under Seiberg dualities, baryonic invariants and even baryonic deformations change simply by a redefinition of their constituent fields [46, 47]. Hence we expect the cascade, and its self-similarity, not to be affected by these operators, which are essentially IR irrelevant at least until the last step of the cascade.

In the present paper we have adopted a pure gauge theory approach to the problem. It would be extremely interesting to study the gravity/string counterpart of the examples we consider.[9] It is interesting to note that the probe approximation seems to be imposed on us by the problem itself. It would be nice to understand from a string theory point of view what is special for a single fractional brane, that cannot be generalized to a bunch $M > 1$ of the same kind of fractional branes. This is of course in a way related to the enlarged flavour group of SU(2) SQCD with respect to SU($N_c > 2$), a fact that to our knowledge has not been really dealt with in the literature on gauge theories from brane setups.

The next issue would be to understand how to generate the extra terms in the superpotential in string theory. Baryonic invariants of a CFT in the AdS/CFT correspondence are usually associated to D3 branes wrapped on 3-cycles [51 – 54]. From this point of view it would seem rather difficult to write deformations of the theory by such operators. However, from the probe brane point of view, the operators are more innocent-looking, quadratic or cubic as they are. Hence we have at least the hope that, as in the usual treatment of flavours [55], deformations involving matter fields can be implemented on the world-volume theory of the probe brane.

We would like to end this discussion with more "phenomenological" issues. It would be very interesting to use the gauge/string correspondence to study the spectrum of the

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[9]See [48 – 50] for the study of other models of DSB from the string/gravity/brane perspective.

theories which display DSB. Again, because of the probe approximation, the study in the string/gravity dual should be along the lines of the one performed for theories with flavours.

We should be able to observe a spectrum characterized by the scale of DSB, and of course the goldstino, i.e. the Goldstone fermion of broken SUSY. Note that we expect the goldstino not to correspond to a bulk (closed string) mode, as the one discussed in [56], but rather to a mode in the probe brane world-volume theory. This does not prevent the existence of other massless modes in the (supersymmetric) bulk, as in the conifold theory (see [57] for the bosonic modes and [56] for their fermionic partners), associated to quantum flat directions in the gauge theory and possibly to Goldstone bosons of broken symmetries. The latter remain of course exactly massless even if SUSY is broken, however we expect in the probe approximation the other modes to remain massless too. In a full back-reacted geometry they would presumably be lifted.

Acknowledgments

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